

Magnetism & matter

Disclaimer : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

Magnetism and matter

Magnetic field lines

Solenoid & Bar magnet

Oscillations of magnetic
dipole

Earth's magnetic field

Magnetic field lines

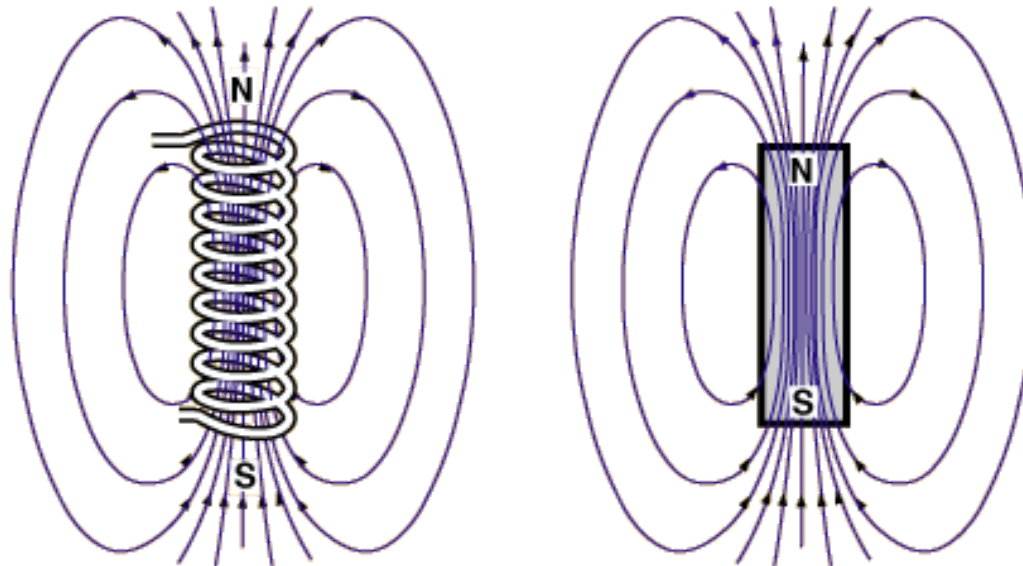
- ❑ Magnetic field lines are imaginary lines drawn along the orientation of a compass
- ❑ Magnetic field lines form continuous closed loops
- ❑ Tangent drawn to the field line at any point indicates the direction of magnetic field at that point
- ❑ Number of field lines passing through a unit surface area (held normal to it) indicates the strength of magnetic field in that region
- ❑ Magnetic field lines do not intersect

This is because if they were to intersect, then at the point of intersection magnetic field would have two different directions simultaneously. This is not physically possible.

- Unlike electric charges, isolated magnetic north and south poles known as *magnetic monopoles do not exist*.
- The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity

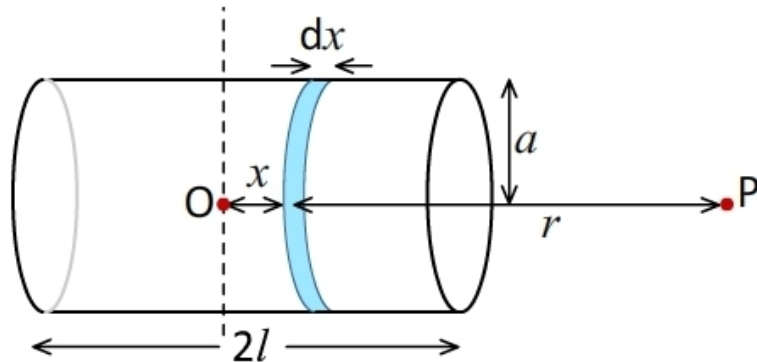
Bar magnet and a solenoid (equivalence)

- ❑ Magnetic field due to a bar magnet and a solenoid at a far axial point (short bar magnet or a solenoid approximation) is the same.
- ❑ Resemblance of magnetic field lines for a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid.



Derivation of B due to a solenoid at a far off axial point

Consider a solenoid of radius R and length $2l$. Let P be a point on the axial line, at a distance r from the centre of the solenoid



dB due to a small segment dx is given by

$$dB = \frac{\mu_o n_o dx i R^2}{2[(r-x)^2 + R^2]^{3/2}}$$

B due to the solenoid is obtained by integrating from $-l$ to $+l$

$$B = \int_{-l}^{+l} \frac{\mu_o n_o dx i R^2}{2[(r-x)^2 + R^2]^{3/2}}$$

$$B = \frac{\mu_o n_o i R^2}{2} \int_{-l}^{+l} \frac{dx}{[(r-x)^2 + R^2]^{3/2}}$$

If $r \gg l$ and $r \gg R$ then the above expression may be approximated to

$$B = \frac{\mu_o n_o i R^2}{2 R^3} \int_{-l}^{+l} dx$$

$$B = \frac{\mu_o n_o i R^2 2l}{2 R^3}$$

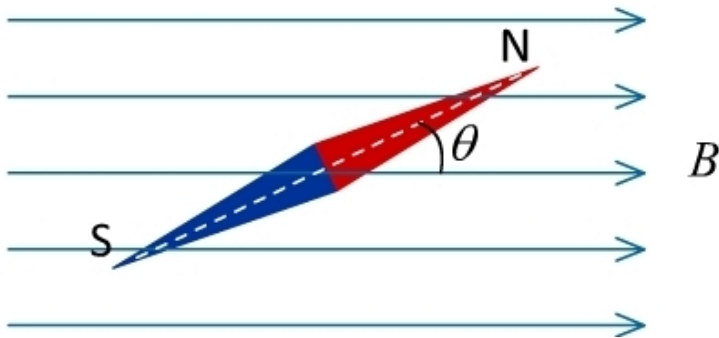
$$B = \frac{\mu_o 2(n_o 2l \pi R^2 i)}{4 \pi R^3}$$

$$B = \frac{\mu_o}{4 \pi} \frac{2m}{R^3}$$

This is similar to B due to a bar magnet of magnetic moment m .

Oscillation of a magnetic dipole in a region of uniform magnetic field

Consider a magnetic dipole of moment $\mathbf{m}$ placed in a region of uniform magnetic field of induction $\mathbf{B}$ at an angle θ .



Restoring torque acting on the dipole is

$$\boldsymbol{\tau} = -\mathbf{m} \times \mathbf{B}$$

$$\tau = -mB \sin(\theta) \quad \text{--- i}$$

Torque acting on a body results in angular acceleration given by

$$\tau = I \alpha \quad \text{--- ii}$$

Comparing equations (i) and (ii)

$$I \alpha = -mB \sin(\theta)$$

$$\alpha = -\frac{mB}{I} \sin(\theta)$$

For small oscillations $\sin(\theta) = \theta$

$$\alpha = -\frac{mB}{I} \theta \quad \text{--- iii}$$

As α is proportional to θ the needle executes SHM. Comparing with relation for acceleration in SHM we get

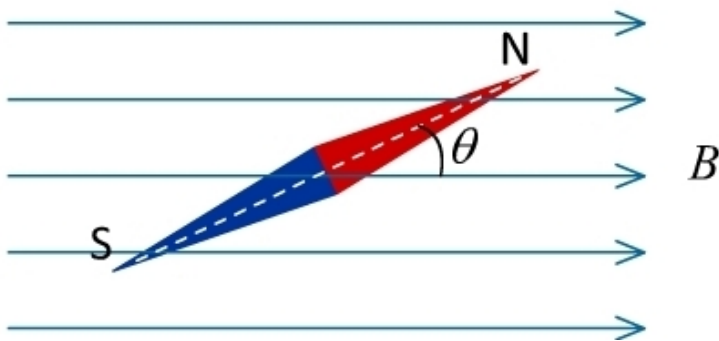
$$\omega^2 = \frac{mB}{I}$$

Using $\omega = 2\pi/T$ we get

$$T = 2\pi \sqrt{\frac{I}{mB}}$$

Potential energy of a magnetic dipole in a region of uniform magnetic field

Consider a magnetic dipole of moment $\mathbf{m}$ placed in a region of uniform magnetic field of induction $\mathbf{B}$ at an angle θ .



Torque acting on the dipole is

$$\tau = -m \times B \sin(\theta) \quad \text{--- i}$$

Work done by external source in causing a small displacement $d\theta$ is

$$dW = mB \sin(\theta) d\theta \quad \text{--- ii}$$

Total work done is obtained by integrating equation (ii)

$$W = -mB \cos(\theta) + C$$

This work done on the dipole is stored as the potential energy in it. Therefore

$$U = -mB \cos(\theta) + C$$

Reference point or orientation can be chosen arbitrarily to take potential energy as zero. Considering initial or reference angle as 90° , the constant of integration becomes zero. Therefore

$$U = -mB \cos(\theta) \quad \text{--- iii}$$

Stable equilibrium

At $\theta = 0^\circ : U = -mB$ (minimum)

Unstable equilibrium

At $\theta = 180^\circ : U = +mB$ (maximum)

Gauss theorem in magnetism

The net magnetic flux through any closed surface is zero.

$$\int_{\text{closed}} \mathbf{B} \cdot d\mathbf{S} = 0$$

- Magnetic field lines are always in closed loops
- Magnetic monopoles (isolated north or south poles) do not exist

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Electrostatics

1. Electric field (E)
2. Electric field lines: open lines from +ve to -ve
3. Electric force (qE)
4. Electric dipole moment ($p = q2d$)
5. Dipole moment vector -ve to +ve
6. Torque on dipole ($\tau = p \times E$)
7. Electric oscillations $T = 2\pi \sqrt{\frac{I}{pE}}$
8. Electric flux $\phi = E \cdot dS$
9. Gauss theorem $\oint E \cdot dS = \frac{q_{\text{enc}}}{\epsilon_0}$

Magnetism

1. Magnetic field (B)
2. Magnetic field lines: closed loops (from N to S outside a bar magnet)
3. Magnetic force ($q \mathbf{v} \times \mathbf{B}$)
4. Magnetic dipole moment ($m = IA$)
5. Dipole moment vector $\perp$ to A
6. Torque on dipole ($\tau = m \times B$)
7. Electric oscillations $T = 2\pi \sqrt{\frac{I}{mB}}$
8. Electric flux $\phi = B \cdot dS$
9. Gauss theorem $\oint B \cdot dS = 0$



Earth's magnetic field

- ❑ Earth's magnetic field is of the order of 10^{-5} T
- ❑ Earth's magnetic field is assumed to arise due to electrical currents produced by convective motion of metallic fluids (consisting mostly of molten iron and nickel) in the outer core of the earth. This is known as the dynamo effect.
- ❑ The north magnetic pole behaves like the south pole of a bar magnet inside the earth and vice versa.
- ❑ Geographic meridian: Vertical plane joining longitude circle and axis of rotation.
- ❑ Magnetic meridian: A vertical plane which pass through the imaginary north pole and south pole.

Earth's magnetic field

- ❑ Magnetic declination: the angle between the true geographic north and the north shown by a compass needle or the angle between geographic meridian and magnetic meridian of the earth
- ❑ Declination is greater at higher latitudes and smaller near the equator .
- ❑ Angle of dip: dip is the angle that the total magnetic field of the earth makes with the surface of the earth

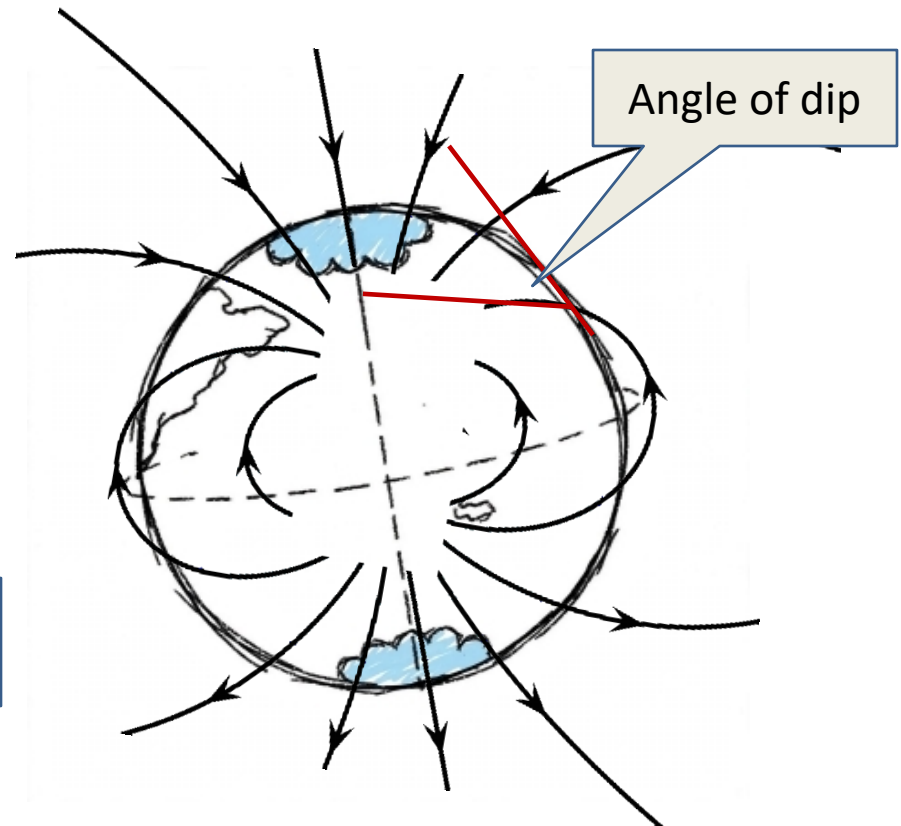
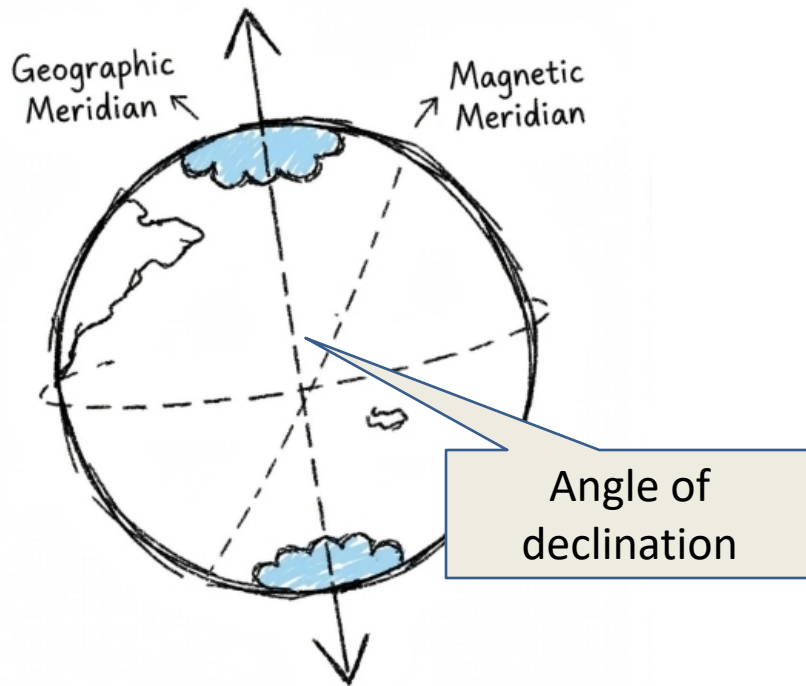
Magnetism & matter

- ❑ If B is the magnetic field and θ is the angle of dip at any point, then these parameters are related as

$$B_H = B \cos(\theta)$$

$$B_V = B \sin(\theta)$$

- ❑ At poles, angle of dip is 90° .



Tangent law

When a magnetic needle is suspended freely in a region of two mutually perpendicular magnetic fields, the needle comes to rest at an angle at which the torque due to one magnetic field exactly balances the torque due to the other magnetic field.

Let one magnetic field (B) be generated by a source such as a current carrying coil or a bar magnet. Let the other magnetic field be the horizontal component of the earth's magnetic field (B_H).

$$\tau_1 = \tau_2$$

$$m B \sin(90^\circ - \theta) = m B_H \sin(\theta)$$

$$B \cos(\theta) = B_H \sin(\theta)$$

$$B = B_H \tan(\theta)$$

